[3]Salas, M., Gumbert, C., and Turkel, E., "Nonunique Solutions to the Transonic Potential Flow Equation," *AIAA Journal*, Vol. 22, No. 1, 1984, pp. 145–146.

[4]Steinhoff, J., and Jameson, A., "Multiple Solutions of the Transonic Potential Flow Equations," *AIAA Journal*, Vol. 20, No. 11, 1982, pp. 1521–1525.

[5]Bauer, F., Garabedian, P., and Chang, I. C., "Wave Drag and the Entropy Inequality in Transonic Flow," *Advances in Computational Transonics* (to be published).

[6]Oswatitsch, K., *Gas Dynamics*, Academic Press, New York, 1956, Chap. 6.

[7]McGrattan, K., "A Comparison of the Potential and the Euler Formulations of the Equations of Transonic Flow," Ph.D. Thesis, New York Univ., New York, NY, 1991.

[8]Thomas, J., and Salas, M., "Far-Field Boundary Conditions for Transonic Lifting Solutions to the Euler Equations," *AIAA Journal*, Vol. 24, No. 7, 1986, pp. 1074–1080.

[9]Bledsoe, M., "The Method of Complex Characteristics for Design of Transonic Blade Sections," Research and Development Rept. DOE/ER/03077-273, New York Univ., New York, NY, June 1986.

High-Velocity Measurements via Laser Doppler Anemometer Using Single- and Multiaxial-Mode Lasers

J.-A. Wang* and C. L. Dancey†
Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061

Introduction

When a multiaxial-mode laser is used in a laser Doppler anemometer (LDA) system the resulting signal contains distinct frequencies other than the Doppler frequency.[1] The other frequencies are the result of the Doppler signal "beating" with the axial modes of the laser. If the beat frequencies are close to the Doppler frequency, it may not be possible to separate them, by electronic filtering, for example. This problem is particularly relevant to high-velocity flows where Doppler frequencies may be comparable to the intermode frequency of the laser. The axial modes and related beat frequencies can be eliminated by installing a tuned etalon to achieve single axial-mode laser operation. Unfortunately, this results in a large loss in incident laser beam power, which is undesirable, particularly for high-velocity measurements.

Although the presence of "beat" frequencies in LDA signals has been experimentally verified, the impact they may have on validated LDA measurements has not been investigated. It is not clear that isolation of the Doppler frequency from the beat frequencies or elimination of the beat frequencies is necessary, in practice. To ascertain the practical effects of multiaxial-mode laser operation on LDA measurements, a sequence of experimental tests were run at relevant flow and LDA operating conditions. From these tests, it is concluded that the presence of the beat frequencies had no significant effect on the measured mean velocities. Data rates were in general higher during multimode operation and measured velocity fluctuation intensities were marginally lower without etalon mode selection.

Experimental Setup

A small-scale (~12× ~15-mm test section) wind tunnel with a nominal test section Mach number of 2 was used for the experiments. The tunnel was small enough to be run from a machine shop air supply. Measurements were made using the 514.5-nm channel of an argon-ion laser in a forward-scatter LDA system with counterbased processing and with 1% comparison tolerance for the rejection of noisy bursts. A single measurement was obtained per Doppler burst, based upon eight fringes with a total of 1024 measurements acquired per sample location. Polystyrene latex spheres of 0.54 μm diameter were used as the seed material for the tests.

The tests consisted of systematically varying the velocity at the LDA measurement volume by traversing the measurement volume along the axis of the tunnel. In this manner, a wide range of mean flow velocities and corresponding Doppler and beat frequencies were obtained. Measurements were made with and without an etalon installed (termed single-mode and multimode operation, respectively) and at two different laser power levels (0.5 and 1.2 W on all lines for single-mode operation; 1.0 and 2.0 W on all lines for multimode operation). Furthermore, a range of electronic filter bandwidths were successively applied at each location to assess the effects of selective filtering on the processed signal. The filter settings include low-limit/high-limit filter combinations of 65/200, 50/200, 20/200, and 10/200 MHz. The etalon was carefully tuned before initiating each set of tests.

Results

For the LDA used in the tests, the Doppler frequency ν_D and the beat frequency ν_B closest to the Doppler frequency vary with the flow velocity as indicated by the solid lines in Fig. 1. (Frequency in megahertz is given on the right axis for convenience.) ν_D and ν_B are widely separated for very low and very high velocities; but they are comparable and in fact pass through one another at a velocity of approximately 250 m/s. (The reference velocity used on the abscissa in both figures is the mean velocity in the tunnel obtained with etalon mode selection, at the maximum power of 1.2 W, and the narrowest permissible bandpass filter combination.) The electronic bandpass filter limits were such that, for multimode operation, at most one beat frequency was possible in the processed signal. All other beat frequencies were easily eliminated by the filters. In many cases, the filters eliminated all beat frequencies.

Figure 1 shows the mean velocity results (left axis) for both single and multimode operation at 1.2 and 2.0 W, respectively. The two groups of data form (within experimental error) two, slightly displaced, parallel lines. It was found that the laser power and, more importantly, filter bandwidth had no significant effect on the measured mean velocities for either group, and in order to simplify the graph these variables were not included in Fig. 1. The small displacement between the two groups of data is not associated directly with the operation of the etalon, but rather it is due to a change in alignment and orientation of the LDA that was required when the etalon was installed and tuned. Accounting for this fixed bias, there was

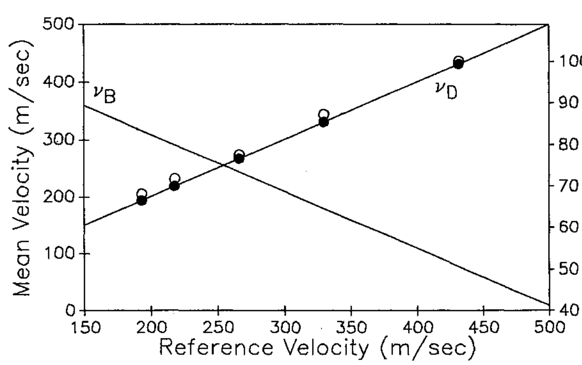

Fig. 1 Measured mean velocity (left axis) and Doppler and beat frequency (right axis) vs tunnel reference velocity, 50/200 MHz bandpass filter limits. ●, 1.2 W, single mode; ○, 2.0 W, multimode.

Received Sept. 24, 1991; revision received Jan. 19, 1992; accepted for publication Feb. 10, 1992.

*Graduate Research Assistant, Department of Mechanical Engineering.

†Assistant Professor, Department of Mechanical Engineering. Member AIAA.

no statistically significant difference between the single and multimode mean velocity results. Furthermore, data rates without the etalon and the laser operation at 2.0 W were typically on the order of several thousand per second compared to less than one thousand per second in the case of single-mode operation at 1.2 W (with comparable photomultiplier voltage and signal conditioner amplification).

The velocity fluctuation intensity (the rms level divided by the measured mean velocity) is shown in Fig. 2 for selected conditions. (Again, for clarity, all of the available data were not plotted in this figure.) For both single and multimode operation, it was found that the measured intensity decreased as the filter bandwidth was tightened about the Doppler frequency and as the laser power was increased. These trends are present in the representative data shown in Fig. 2. This behavior was expected.

Comparing single and multimode operation, the single-mode intensities generally showed more sensitivity to laser power. More significantly, the multimode intensities were generally lower than those obtained with tuned etalon, single-mode selection, although the difference between the two was frequently only slightly beyond the limits for statistical significance. These results were not expected.

Dopheide and Durst[1] demonstrated (using a He-Ne laser) that LDA signals containing several beat frequencies have the appearance of noisy signals. One would expect such signals to yield larger "effective" velocity fluctuation intensities and/or reduced data rates due to the presence of additional zero crossings in the high-passed signal. Neither was found to occur in the reported tests, particularly when the additional available laser power was utilized for the multimode tests.

A better understanding and explanation of these test results would require more extensive and systematic testing; however a speculative explanation is proposed: For multimode operation, part of the "noise" in the LDA signal is really the superposition of many higher-frequency beat signals with the Doppler frequency. Theoretically, and other things being equal, single-mode operation should result in an increase in the signal-to-noise ratio (SNR). However, for the reported tests the theoretical gain in SNR may have been entirely offset by the attendant decrease in available laser power. Since the powers reported here were measured on all transmitted wavelengths rather than on the 514.5-nm wavelength alone, and since the power available in the 514.5-nm line depends upon which axial mode is selected during etalon tuning, it is likely that the actual power available was significantly lower for the single-mode tests. In addition, spectral analysis[1] shows that the power in the beat frequencies is lower than the power at the Doppler frequency and, in the present case, many beat frequencies did not contribute to the "noise" since electronic filtering conveniently removed all but one. As a consequence, it is possible that the actual SNR was higher in multimode tests at 1.0 and 2.0 W than in the single-mode tests at 1.2 W. This would be consistent with the improved data rate and lower velocity fluctuation intensities found using the multiaxial-mode laser.

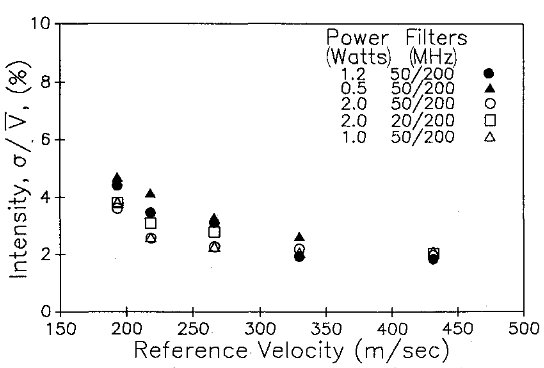

Fig. 2 Measured turbulence intensity (%) vs tunnel reference velocity. Open symbols, multimode operation; solid symbols, single-mode operation.

Conclusions

The results of these tests indicate that the presence of beat frequencies in the LDA signal does not necessarily result in poor quality measurements or reduced data rate. In particular, no effect was observed in the mean velocity measurements, and the data rates for multimode operation exceeded single-mode data rates when the additional beam power was employed. Higher-level velocity fluctuation intensities were not associated with multimode operation.

Reference

[1]Dopheide, D., and Durst, F., "High Frequency Laser Doppler Measurements Using Multiaxial-Mode Lasers," *Applied Optics*, Vol. 20, No. 9, 1981, pp. 1557–1570.

Induced Drag of a Wing in a Circular Wind Tunnel

Yuzo Yamamoto*
Gifu National College of Technology, Motosu, Gifu, Japan

Nomenclature

b = wing span
E = complete elliptic integral of the second kind, modulus k
e = span efficiency factor, $C_{Di} = C_L^2/(\pi A_R e)$
K = complete elliptic integral of the first kind, modulus k
k = modulus of elliptic function, $\lambda^2 = (b/d)^2$
sn v, cn v, dn v = Jacobian elliptic functions, modulus k
v = variable, modulus k, x = sn v
W = uniform downwash
w = downwash on wing vortex trace
$Z(v)$ = Jacobian theta function, modulus k, $E(v) - (E/K)v$
Γ = bound vortex circulation
γ = trailing vortex strength
ϵ = clearance between wing tip and wind-tunnel boundary to wind-tunnel radius ratio, $1 - (b/d)$

Subscripts
c = closed-tunnel case
0 = open-tunnel case

Introduction

IN our previous paper,[1] the exact expression of the spanwise optimum lift distribution and the minimum induced drag of a straight wing in a circular closed wind tunnel have been obtained from the Trefftz plane flowfield analysis. In this study, the spanwise optimum lift distribution and minimum induced drag of a wing in a circular wind tunnel, with either a closed or an open working section, are examined exactly. In the closed-tunnel case, an alternative analysis with Ref. 1 is derived, and the lift distribution and the induced drag that are obtained by the present analysis are coincident with those of Ref. 1. In the Trefftz plane flowfield analysis, the method of

Received April 8, 1991; revision received Sept. 5, 1991; accepted for publication Oct. 20, 1991.

*Professor, Department of Mechanical Engineering.